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Design of Cooperating Spherical *RPR* Robots

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Abstract

This paper uses the kinematic mapping into the image space of spherical displacements to design a cooperating spherical robot system for workpiece orientating. The kinematic chain formed by the cooperating robots grasping the workpiece forms a multi degree of freedom closed chain which is also known as a robotic mechanism. The spherical robots considered are spherical RPR open chains and they rigidly grasp the workpiece to form a 3 degree of freedom closed chain. The design problem considered is to determine the base locations and grasp points that enable the cooperating robots to guide a workpiece through an arbitrary number of desired orientations. An example of the design of a cooperating spherical RPR robot system for six(6) desired orientations is presented.

Keywords cooperating spherical robots, spherical mechanisms, robotic mechanisms, rigid body guidance, approximate motion synthesis.

1 Introduction

This paper presents the design of cooperating spherical RPR robots for approximate rigid body motion. The cooperating robots may also be viewed as a three(3) degree of freedom spherical robotic mechanism. A spherical robotic mechanism is a multi degree of freedom simple spherical closed kinematic chain. In the case of 3-*dof* spherical robotic mechanisms the closed chain consists of two RPR spherical open chains, or triads, which connect the workpiece, or floating link, to the ground.

For facilitating the kinematic synthesis of the 3-*dof* spherical robotic mechanism we view its RPR spherical open chains as variable crank length spherical RR dyads and employ well known dyadic synthesis techniques for rigid body guidance, see Suh and Radcliffe 1978. The variable link length spherical RR dyad can be realized, and manufactured, by using a RRR open chain, see Fig. 1 and Larochelle 1994. To maximize the workspace of the mechanism the two links in each of the $3R$ open chains are chosen to be 90(deg) in length, see Ouerfelli and Kumar 1991.

In Bodduluri 1991 the solution to four position rigid body guidance for the spatial $4C$ robotic mechanism was presented and in Larochelle and McCarthy 1994 a design procedure for an arbitrary number of prescribed positions was demonstrated. Here we extend the works of Ravani and Roth 1983, Bodduluri 1990, and Larochelle 1994, and Larochelle and McCarthy 1994 to the dimensional synthesis of cooperating spherical robots for n position rigid body guidance, see Fig. 1. The first step of the design process is to define the design goal of the robotic system in terms of the desired positions of the workpiece and to specify the bounds on the crank lengths of the robotic mechanism, i.e. limits for minimum and maximum spherical translation of the P joint of each robot. Note that if no bounds are placed upon the motion at the P joints that the system can reach any desired orientation [1]. However, manufacture of such a spherical robot is not possible. Moreover, in the extreme case that the two crank lengths are held fixed we have a spherical $4R$ mechanism which in general can guide a body exactly through only five positions, see Suh and Radcliffe 1978. Therefore, for an arbitrary finite number of desired workpiece orientations we utilize an optimization procedure first derived by Ravani and Roth 1983 by which we vary the synthesis variables so as to minimize the position error of the system through an arbitrary number of orientations for specified bounds on the motion of the P joints.

The optimization algorithm employed involves

[1] This can be seen by viewing each RPR robot as an ideal $3R$ robot wrist.

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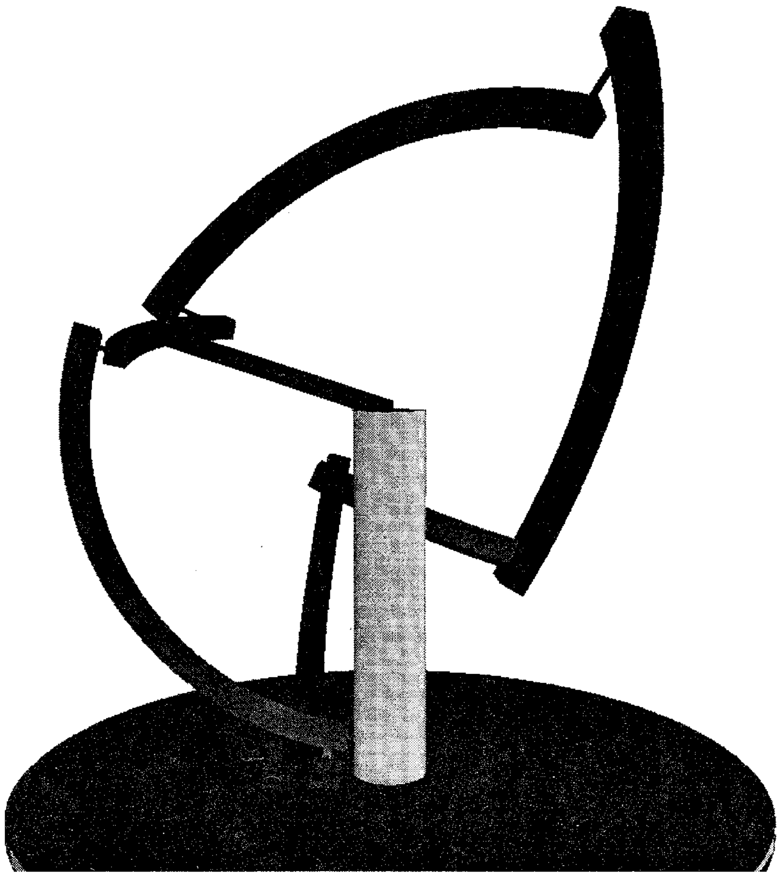

Figure 1: Cooperating Spherical RPR Robots Illustrated as Cooperating $3R$ Robots

writing the kinematic constraint equation of the variable crank length spherical RR dyad using the components of a quaternion. We view these equations as constraint manifolds in the image space of spherical displacements, see Bottema and Roth 1979 and McCarthy 1990. The result is an analytical representation of the workspace of the dyad which is parameterized by its dimensional synthesis variables. We then combine two variable crank length spherical RR dyads to form a $4R$ closed chain. The constraint manifold of the variable crank length $4R$ mechanism is simply the intersection of the constraint manifolds of its two RR subchains. This intersection provides an analytical representation of the workspace of the cooperating robot system in the image space of spherical displacements. The optimization goal is to determine the design variables such that all of the prescribed positions are either: (1) in the workspace, or, (2) the workspace comes as close as possible to all of the desired positions subject to the constraints on the variable crank length. The result of the design process is a cooperating spherical robot system that guides the workpiece as close as possible to the desired orientations subject to the imposed P joint translation limits. A design case study for 6 desired orientations is presented.

2 Spherical Displacements

A general spherical displacement may be described by a 3×3 orthonormal rotation matrix $[A]$. Using the rotation axis $\mathbf{s}$ and the rotation angle θ associated with $[A]$ we can represent the spherical displacement by the four dimensional vector, $\mathbf{q}$, which is written as, see McCarthy 1990,

$$\begin{aligned} q_1 &= s_x \sin\frac{\theta}{2} \\ q_2 &= s_y \sin\frac{\theta}{2} \\ q_3 &= s_z \sin\frac{\theta}{2} \\ q_4 &= \cos\frac{\theta}{2} \end{aligned} \tag{1}$$

We refer to $\mathbf{q}$ as a quaternion. The components of $\mathbf{q}$ satisfy,

$$G_s(\mathbf{q}) \quad : \quad q_1^2 + q_2^2 + q_3^2 + q_4^2 - 1 = 0 \tag{2}$$

and lie on a unit hypersphere which we denote as *the image space of spherical displacements.*

The rotation matrix, $[A]$ can be recovered from the quaternion, $\mathbf{q}$, as follows,

$$[A] = \tag{3}$$

$$\begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 - q_3q_4) & 2(q_1q_3 + q_2q_4) \\ 2(q_1q_2 + q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 - q_1q_4) \\ 2(q_1q_3 - q_2q_4) & 2(q_2q_3 + q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix}$$

3 Constraint Manifolds

In this section we derive the algebraic constraint manifold of the spherical RR dyad. The constraint manifold is derived by using the geometric conditions that the joints of the dyad impose on the moving body. The vector equations for the geometric constraints are based upon the work of Suh and Radcliffe 1978, and Bodduluri 1990.

A spherical RR dyad is shown in Fig. 2. Let the axis of the fixed joint, or the location of the base of the RPR robot, be specified by the vector $\mathbf{u}$ measured in the fixed reference frame F and let the moving axis, or the location of the grasp point of the RPR robot, be specified by $\boldsymbol{\lambda}$ measured in the moving frame M. Because the two axes are connected by a rigid link the angle between the two axes of the dyad remains constant and we have,

$$\mathbf{u} \cdot [A]\boldsymbol{\lambda} = \cos\alpha \tag{4}$$

To obtain an algebraic expression for the constraint manifold in the image space of spherical displacements we substitute Eq. 3 into Eq. 4,

$$RR_{sph}(\mathbf{q}, \mathbf{r}) : \qquad \mathbf{u} \cdot [A(\mathbf{q})]\boldsymbol{\lambda} - \cos\alpha = 0 \tag{5}$$

Given $\mathbf{u}$, $\boldsymbol{\lambda}$, and α the constraint manifold of that dyad is the set of all image points, $\mathbf{q}$, that are solutions to Eq. 5 and it represents all possible locations of M with respect to F for the dyad.

4 Fitting Image Curves

We now describe the method presented by Ravani and Roth 1983 to perform dimensional synthesis using constraint manifolds. The first step is to formulate the constraint manifold of the kinematic chain, $CM(\mathbf{q}, \mathbf{r})$.

$$CM(\mathbf{q}, \mathbf{r}) : \begin{pmatrix} RR_{sph-a}(\mathbf{q}, \mathbf{r}) \\ RR_{sph-b}(\mathbf{q}, \mathbf{r}) \\ G_{sph}(\mathbf{q}) \end{pmatrix} = \mathbf{0} \tag{6}$$

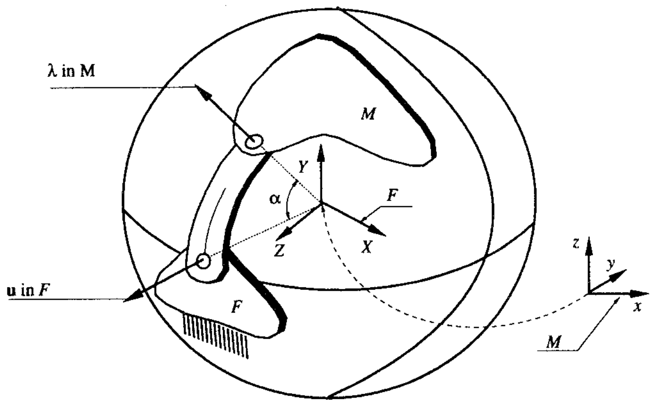

Figure 2: A Spherical RR Dyad

where $RR_{sph-a}(\mathbf{q},\mathbf{r})$ and $RR_{sph-b}(\mathbf{q},\mathbf{r})$ are from Eq. 5 written for each dyad of the chain, $G_s(\mathbf{q})$ is the quaternion constraint equation, Eq. 2, and $\mathbf{r}$ is the vector of dimensional synthesis variables for both dyads.

The goal is to determine the design variables $\mathbf{r}$ such that the constraint manifold passes through, or as close as possible to, the n desired points in the image space. Let $\mathbf{q}_d$ represent one desired point in the image space. We assume that $\mathbf{q}_d$ does not lie on the constraint manifold and write a Taylor series expansion of the constraint manifold about $\mathbf{q}_d$.

$$CM(\mathbf{q}_d,\mathbf{r}) + \frac{\partial CM(\mathbf{q}_d,\mathbf{r})}{\partial \mathbf{q}_d}(\mathbf{q}-\mathbf{q}_d) = 0 \qquad (7)$$

Let us now reformulate Eq. 7 as a system of linear equations,

$$[J]\mathbf{x} = \mathbf{b} \qquad (8)$$

where, $[J]$ is the matrix of partial derivatives of $CM(\mathbf{q},\mathbf{r})$, $\mathbf{b} = -CM(\mathbf{q}_d,\mathbf{r})$, and $\mathbf{x} = \mathbf{q} - \mathbf{q}_d$. In general there will be infinite solutions to Eq. 8.

The minimum norm solution of Eq. 8 is found by use of the pseudo inverse of $[J]$,

$$[J]^+ = [J]^T([J][J]^T)^{-1} \qquad (9)$$

which yields,

$$\mathbf{x}^* = [J]^+\mathbf{b} = \left\{[J]^T([J][J]^T)^{-1}\right\}\mathbf{b} \qquad (10)$$

From the definition of $\mathbf{x}$ we have,

$$\mathbf{q}^* = \mathbf{q}_d + \mathbf{x}^* \qquad (11)$$

where $\mathbf{q}^*$ approximates the point on the constraint manifold closest to $\mathbf{q}_d$. Moreover, we may use $\mathbf{q}^*$ to approximate the normal distance, $e(\mathbf{r})$, from $\mathbf{q}_d$ to the constraint manifold,

$$e(\mathbf{r})^2 = (\mathbf{q}^* - \mathbf{q}_d)^T(\mathbf{q}^* - \mathbf{q}_d) \qquad (12)$$

Finally, performing n position synthesis requires computing $e(\mathbf{r})$ for each desired position $\mathbf{q}_d$. The total error is then given by $E(\mathbf{r}) = \sum_{i=1}^{n} e_i^2(\mathbf{r})$ Thus, we have formulated the n position dimensional synthesis problem in the form of a nonlinear minimization problem with objective function $E(\mathbf{r})$. For further discussion of constraint manifold fitting see Larochelle 1994, Bodduluri 1990, and Ravani and Roth 1983.

5 Case Study

We now present an example of the design of the cooperating spherical RPR robot system for 6 position rigid body guidance.

The 6 desired positions are listed in Tbl. 1. The 12 element design vector $\mathbf{r}$ is,

$$\mathbf{r} = \begin{bmatrix} \mathbf{u}_a \\ \boldsymbol{\lambda}_a \\ \mathbf{u}_b \\ \boldsymbol{\lambda}_b \end{bmatrix} \qquad (13)$$

where $\mathbf{u}$ and $\boldsymbol{\lambda}$ are the base location and grasp points of each RPR robot and the subscripts a and b denote which robot the quantity is associated with. Note that the variable crank lengths are not design variables to be sent to the optimization routine since we allow α_a and α_b to vary during the motion of the robotic system. In this example we have constrained α_a and α_b such that,

$$75.0(\text{deg}) \le \alpha_a \le 85(\text{deg})$$
$$75.0(\text{deg}) \le \alpha_b \le 85(\text{deg})$$

The variable crank lengths are incorporated into the design procedure as follows.

1. Formulate an initial guess for the design vector from a spherical $4R$ mechanism which is a solution to three of the n desired positions.
2. Using the guess for the base locations and grasp points solve for the two crank lengths such that the mechanism passes through the first position using the crank constraint equation, Eq. 4.
3. For each dyad, if the crank length is less than its lower bound then set it equal to its lower bound, if it is greater than its upper bound then set it equal to its upper bound.
4. Evaluate the error for this design to the first position.

Pos.	Long.	Lat.	Roll
1	−25.05	23.16	−40.09
2	30.00	20.00	15.00
3	45.00	25.00	20.00
4	60.00	30.00	25.00
5	75.00	30.00	20.00
6	90.00	30.00	0.00

Table 1: 6 Desired Positions

Pos.	DRIVING	DRIVEN	Error
1	84.99	84.96	$4.29E-14$
2	75.02	85.00	$6.10E-14$
3	78.22	84.46	$2.08E-14$
4	84.99	84.90	$2.11E-15$
5	83.65	82.33	$5.68E-14$
6	75.00	85.00	$4.97E-16$
			$\sum = 1.84E-13$

Table 2: Position Results

5. Repeat steps $2-4$ for each of the n desired positions.
6. Send the design vector and the n position errors to the optimization routine.
7. The result of the optimization routine is a better guess to the design vector. With the new design vector repeat steps $2-6$ until the algorithm has converged to a solution. If the total error is acceptable then the design is completed. If not, then select a new grouping of three of the n positions and repeat steps $1-7$.

The results of the optimization procedure are the base locations and grasp points of the robots as well as the required crank lengths, or P joint translations. The design vector provides the location($\mathbf{u}_a$,$\mathbf{u}_b$) of the base of each robot and their corresponding grasp points on the workpiece, see Tbl. 3. In Tbl. 2 the resulting error at each position and the required crank lengths are listed. The final cooperating spherical RPR robot system is shown in Fig. 3 with the workpiece in desired orientation #1.

6 Conclusion

In this paper we have presented our development of an algorithm, for the dimensional synthesis of two cooperating spherical RPR robots for workpiece orientating. Specifically, we synthesize the robot system

Design Vector $\mathbf{r}$	
Initial Guess	Final Design
−0.72233	−0.86620
0.47945	0.20586
0.49837	0.45532
−0.80591	−0.61026
−0.16287	−0.59043
−0.56919	−0.52819
−0.95856	−0.90968
0.19023	0.31613
0.21206	−0.26934
−0.65721	−0.06470
−0.43955	−0.56649
−0.61227	−0.82153

Table 3: Optimization Results

to guide a body through an arbitrary number of orientations subject to its P joint translation bounds. The synthesis procedure utilizes an algebraic formulation for the constraint manifold of the spherical RR dyad to define the workspace of the system in the image space of spherical displacements. The result of the optimization is the base location and grasp points for each each robot as well as the P joint translation in each of the desired orientations. Future work will address the stiffness and force transmission characteristics of the cooperating robot system presented here. Our hope is that such systems will prove useful when utilized as robot wrists by resulting in robot wrists which have increased payload capacity and stiffness.

7 Acknowledgments

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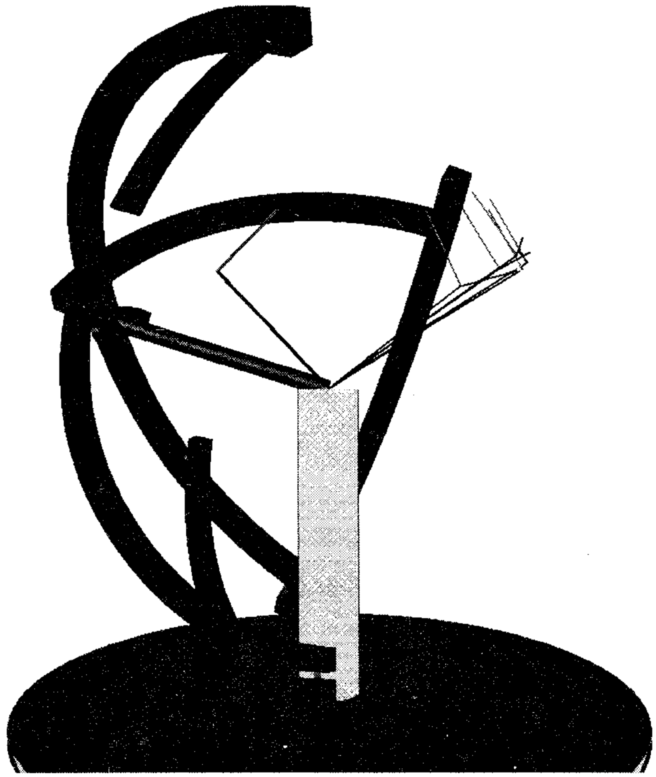

Figure 3: Two Cooperating Spherical RPR Robots

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